TRIGONOMETRY

Trigonometry is a branch of mathematics that deals with relationship (s) between angles and sides **of triangles.**

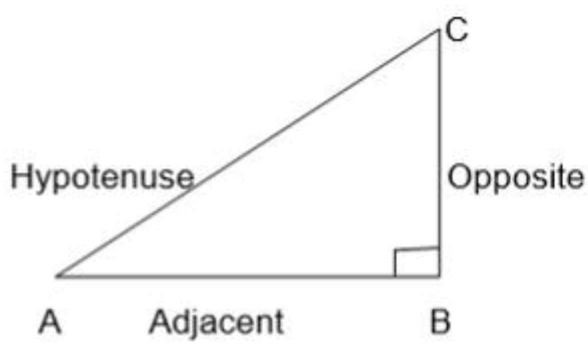
Trigonometric Ratios

The Sine, Cosine and Tangent of an Angle Measured in the Clockwise and Anticlockwise Directions

Determine the sine, cosine and tangent of an angle measured in the clockwise and anticlockwise directions

The basic three trigonometrical ratios are sine, cosine and tangent which are written in short as Sin, Cos, and tan respectively.

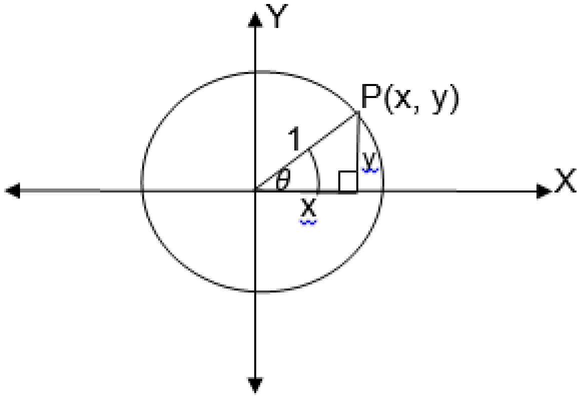
Consider the following right angled triangle.



From the figure above, $\sin A = \frac{Opposite}{Hypotenus}$

$\cos A = \frac{Adjacent}{Hypotenuse}$ and $\tan A = \frac{Opposite}{Adjancent}$

Also we can define the above triangle ratios by using a unit Circle centered at the origin.



$\text{Sin}\,\theta = y$

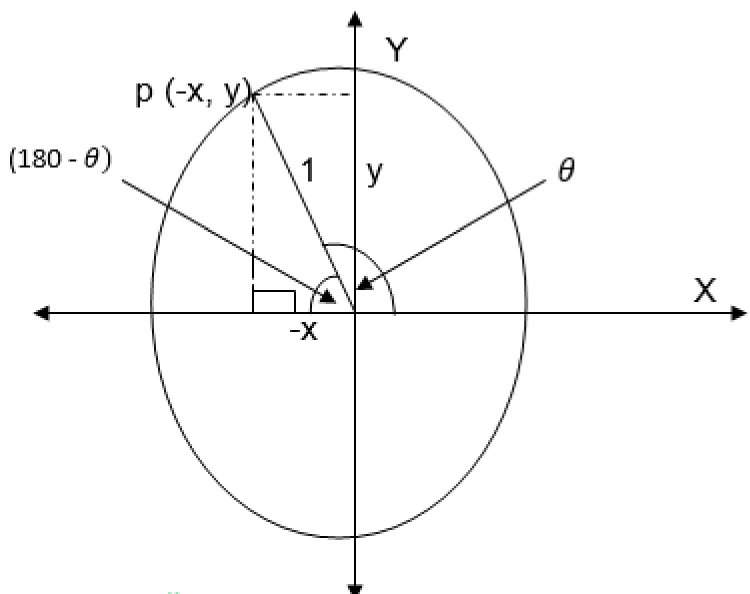
$\text{Cos}\,\theta = x$

$\text{Tan}\,\theta = \frac{y}{x}$

Note that the angle θ is acute ($0 < \theta < 90^0$) and the point P(x, y) is on the unit circle

If θis an obtuse angle (90^0<θ<180 style="box-sizing: border-box;" sup="">0

) then the trigonometrical ratios are the same as the trigonometrical ratio of 180^0-θ



$\text{Sin}\,\theta = \sin(180^0 - \theta) = \frac{y}{1} = y$

So $\sin\theta = y$

$\text{Cos}\,\theta = \text{Cos}\,(180^0 - \theta) = \frac{-x}{1} = -x$

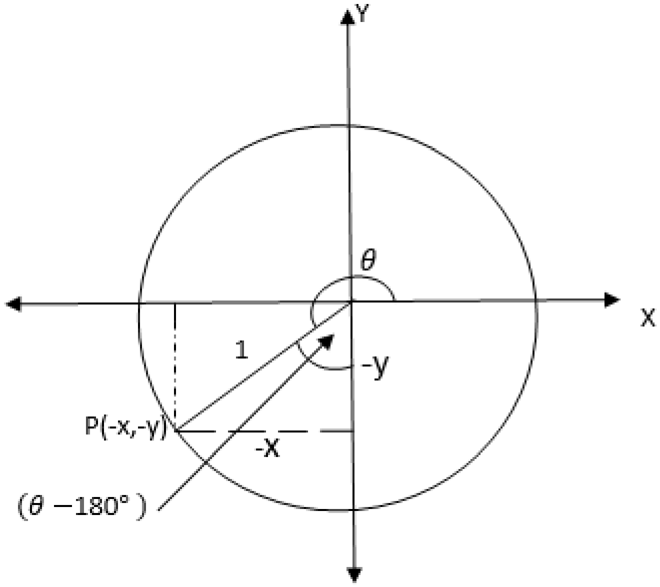
So $\cos\theta = -x$

$\text{Tan}\,\theta = \text{Tan}\,(180^0 - \theta) = \frac{y}{-x}$

So $\tan\theta = \frac{-y}{x}$

If θis a reflex angle (180^0< θ<270 style="box-sizing: border-box;" sup="">0

) then the trigonometrical ratios are the same as that of θ- 180^0



$\text{Sin } \theta = \sin(\theta - 180^0) = \frac{-y}{1} = -y$

So $\text{Sin } \theta = -y$

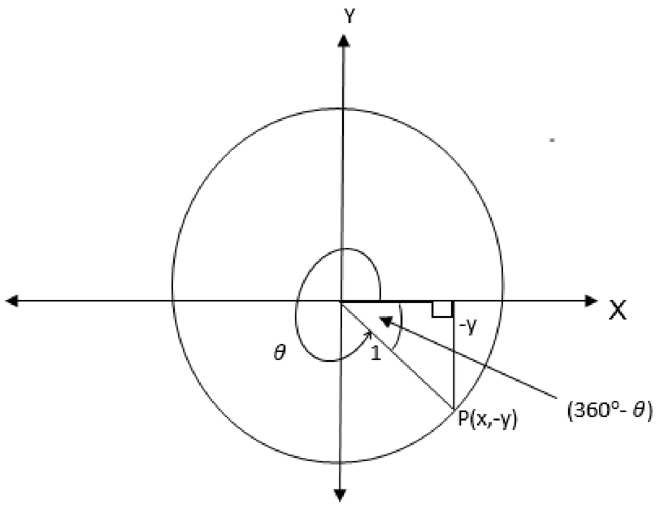
$\text{Cos } \theta = Cos(\theta - 180^0) = \frac{-x}{1} = -x$

$\text{Cos } \theta = -x$

$\text{Tan } \theta = \text{Tan } (\theta - 180^0) = \frac{-y}{-x} = \frac{y}{x}$

$\text{Tan } \theta = \frac{y}{x}$

If θis a reflex angle ($270^0 < \theta < 360^0$), then the trigonometrical ratios are the same as that of $360^0 - \theta$



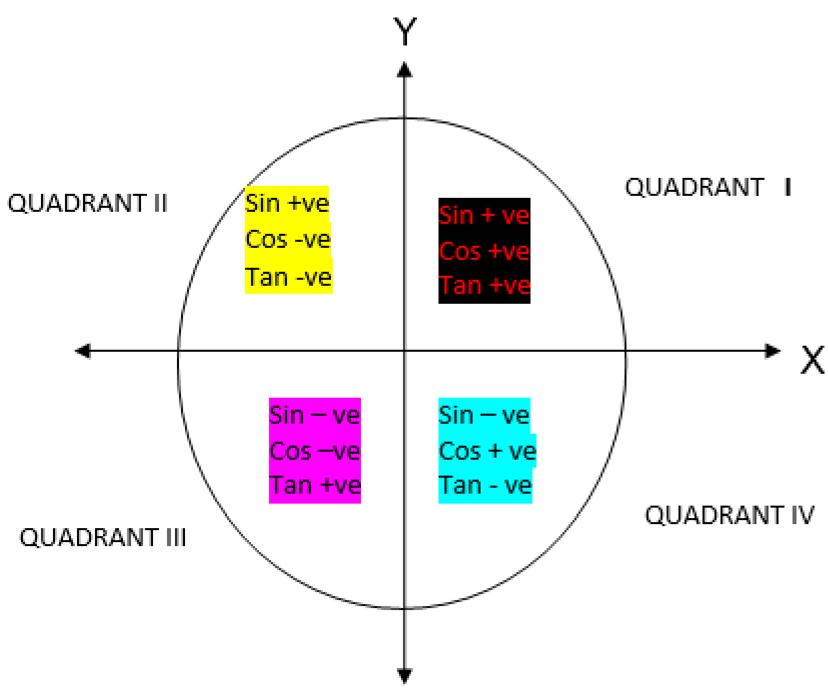
Sin θ = Sin ($360^0 - \theta$) $= \frac{-y}{x}$ = -y, that is Sin θ = -y

Cosθ = Cos ($360^0 - \theta$) = $\frac{x}{1}$ = x, that is Cos θ = x

And Tan θ = Tan ($360^0 - \theta$) = $\frac{-y}{x}$

We have seen that trigonometrical ratios are positive or negative depending on the size of the angle and the quadrant in which it is found.

The result can be summarized by using the following diagram.



Trigonometric Ratios to Solve Problems in Daily Life

Apply trigonometric ratios to solve problems in daily life

Example 1

Write the signs of the following ratios

a. Sin 170^0

b. Cos 240^0

c. Tan 310^0

d. sin 30^0

Solution

a)Sin 170^0

Since 170^0 is in the second quadrant, then Sin 170^0 = Sin (180^0-170^0) = Sin 10^0

∴Sin 170^0 = Sin 10^0

b) Cos 240^0 = -Cos $(240^0-180^{0)}$= -Cos 60^0

Therefore Cos 240^0= -Cos 60^0

c) Tan 310^0 = -Tan (360^0-310^0) = - Tan 50^0

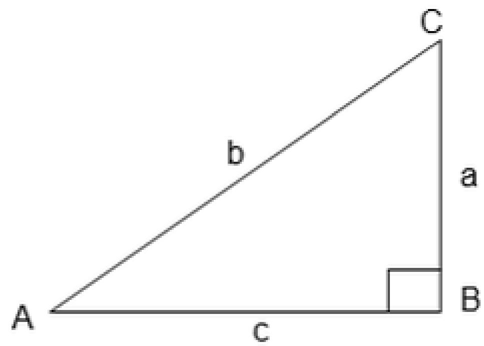
Therefore Tan 310^0= -Tan 50^0

d) Sin 300^0= -sin (360^0-300^0) = -sin 60^0

Therefore sin 300^0= - Sin 60^0

Relationship between Trigonometrical ratios

Consider Δ ABC



Angles A and C are complementary, that is $A+C = 90^0$

Therefore $C = 90^0 - A$

But $\text{Sin A} = \frac{a}{b}$ and $\text{Cos C} = \frac{a}{b}$, then $\text{SinA} = \text{CosC} = \frac{a}{b}$

$\therefore \text{SinA} = \text{Cos}(90^0 - A)$

The above relationship shows that the Sine of angle is equal to the cosine of its complement.

Also from the triangle ABC above

$\text{Sin A} = \frac{a}{b}$ and $\text{Cos A} = \frac{c}{b}$ while $\text{Tan A} = \frac{a}{c}$, now $\frac{\text{Sin A}}{\text{Cos A}} = \frac{a}{b} \div \frac{c}{b} = \frac{a}{b}$

$\frac{\text{Sin A}}{\text{Cos A}} = \frac{a}{c}$, but $\frac{a}{b}$ = Tan A

$$\therefore \quad \boxed{\text{Tan A} = \frac{\text{Sin A}}{\text{Cos A}}}$$

Again using the ΔABC

$b^2 = a^2 + c^2$ (Pythagoras theorem)

And

$$\text{Sin}^2 A + \text{Cos}^2 A = \left(\frac{a}{b}\right)^2 + \left(\frac{c}{b}\right)^2$$

$$\text{Sin}^2 A + \text{Cos}^2 A = = \frac{a^2}{b^2} + \frac{c^2}{b^2}$$

$$\text{Sin}^2 A + \text{Cos}^2 A = \frac{a^2+c^2}{b^2}$$

But $a^2+c^2 = b^2$

$$\text{Sin}^2 A + \text{Cos}^2 A = \frac{b^2}{b^2} = 1$$

$$\therefore \boxed{\text{Sin}^2 A + \text{Cos}^2 A = 1}$$

Example 2

Given thatA is an acute angle and Cos A= 0.8, find

a. Sin A

b. tan A.

Solution:

a) $\text{Cos } A = 0.8 = \frac{8}{10} = \frac{4}{5}$

From $\cos^2 A + \text{Sin}^2 A = 1$

$$\left(\frac{4}{5}\right)^2 + \text{Sin}^2 A = 1$$

$$\text{Sin}^2 A = 1 - \left(\frac{4}{5}\right)^2$$

$$\text{Sin}^2 A = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\text{Sin A}= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\therefore \text{Sin A}= \frac{3}{5}$$

(b) $\tan A = \frac{\text{Sin A}}{\text{Cos A}} = \frac{3}{5} \div \frac{4}{5}$

$$= \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$$

$$\therefore \tan A = \frac{3}{4}$$

Example 3

If A and B are complementary angles,

find Cos B if Sin A= $\frac{1}{8}$

Solution

If A and B are complementary angle

Then Sin A = Cos B and Sin B = Cos A

Now Sin A = $\frac{1}{8}$ = Cos B.

$\therefore$Cos B = $\frac{1}{8}$

Example 4

Given that θand βare acute angles such that $\theta + \beta = 90^0$ and Sinθ= 0.6, find tanβ

Solution

$$\text{Tan } \beta = \frac{Sin\beta}{Cos\beta} \text{ .... } (i)$$

And Sin θ = Cos β = 0.6 because θ and β re complementary angles

Now

$$\text{Sin } \beta = \sqrt{1 - \text{Cos}^2\beta}$$

$$\text{Sin } \beta = \sqrt{1 - 0.6^2}$$

$$= \sqrt{1 - \left(\frac{6}{10}\right)^2}$$

$$= \sqrt{1 - \frac{36}{100}}$$

$$= \sqrt{\frac{64}{100}} = \frac{8}{10}$$

$$\text{Sin } \beta = \frac{8}{10}$$

$$\tan \beta = \frac{Sin\beta}{Cos} = \frac{8}{10} \div 0.6$$

$$= \frac{8}{10} \div \frac{6}{10} = \frac{8}{10} \times \frac{10}{6}$$

$$\therefore \tan \beta = \frac{4}{3}$$

Exercise 1

For practice

1. Given that Cos $\theta = \frac{4}{5}$ find sinθ

2. If Sinα = $\frac{\sqrt{2}}{5}$ (find Sin (90^0 - α)

3. Without using tables, find tan A if A is an acute angle and Cos A= 0.225.

4. If Cos θ = $\frac{p}{q}$ and Tan θ = r/p Find Sin θ

5. If tanB = $\frac{k}{\sqrt{u}}$ and CosB = $\frac{\sqrt{u}}{m}$

   Show that m= $\sqrt{k^2 + u}$

Sine and Cosine Functions

Sines and Cosines of Angles 0 Such That -720°≤θ≥ 720°

Find sines and cosines of angles 0 such that -720°≤θ≥ 720°

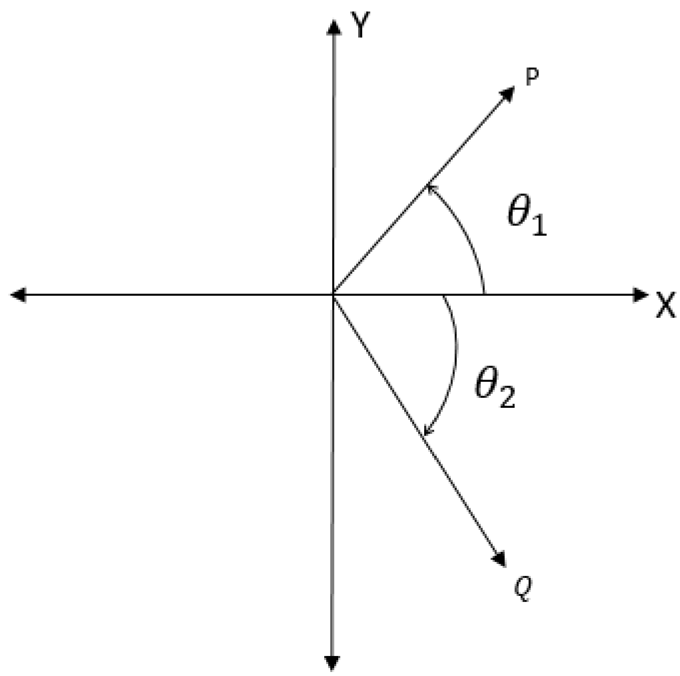
Positive and Negative angles

An angle can be either positive or negative.

Definition:

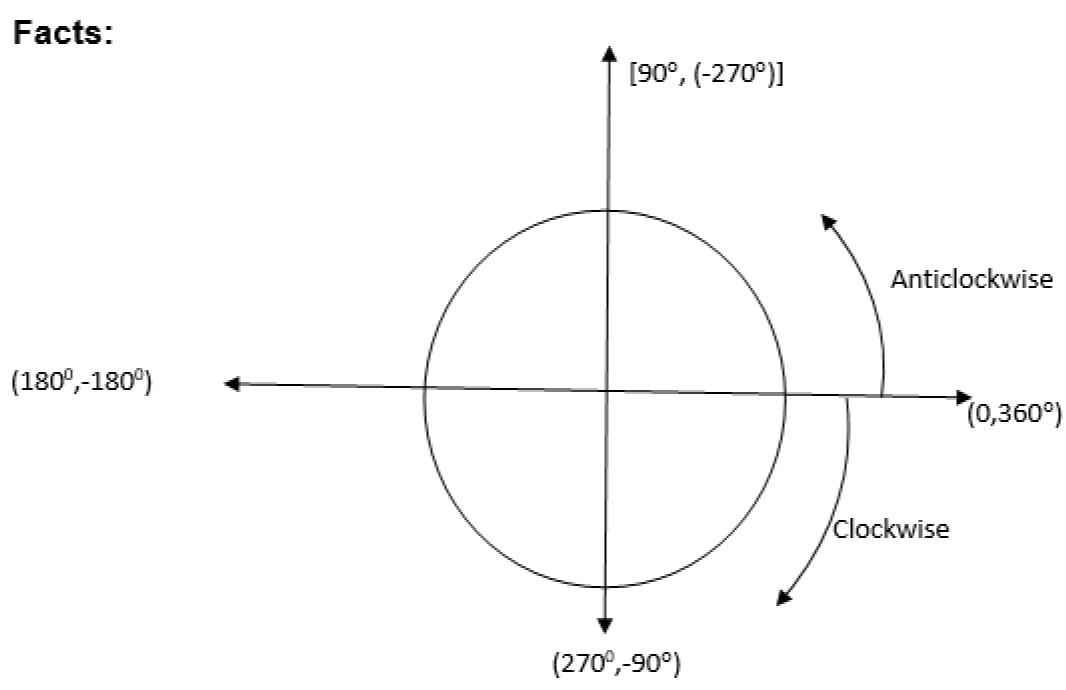
Positive angle: is an angle measures in anticlockwise direction from the positive X- axis

Negative angle: is an angle measured in clockwise direction from the positive X-axis



From the figure above, θ_1 is positive while θ_2 is negative.

Facts:



a. From the above figure if is a positive angle then the corresponding negative angle to is (-360^0) or (+ - 360^0)

b. .If is a negative angle, its corresponding positive angle is (360+)

Example 5

Find thecorresponding negative angle to the angle θif ;

a. $\theta = 58^0$

b. $\theta = 245^0$

Solution:

a) $\theta = 58^0$

$\theta = -360^0 + \theta = \theta - 360^0$

$\theta = 58 - 360^0 = -302^0$

The corresponding negative angle to angle θ is -302^0

b) $\theta = 245^0$

$\theta = 245^0 = 245^0 - 360^0$

$\theta = -115^0$

∴The corresponding angle to the $\theta = 245^0$ is -115^0

Example 6

What is the positive angle corresponding to - 46°?

***Solution*:**

If θ is negative, then its corresponding positive angle is $\theta + 360^0$

So $-460^0 + 360^0 = 314^0$

∴ -460^0 corresponds to 314^0

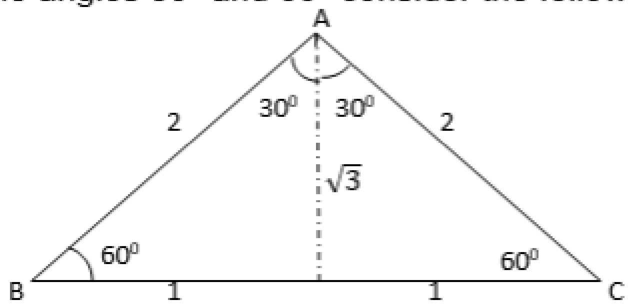
SPECIAL ANGLES

The angles included in this group are 0^0, 30^0, 45^0, 60^0, 90^0, 180^0, 270^0, and 360^0

Because the angle 0^0, 90^0, 180^0, 270^0, and 360^0, lie on the axes then theirtrigonometrical ratios are summarized in the following table.

ANGLE	0^0	90^0	180^0	270^0	360^0
Sine	0	1	0	-1	0
Cosine	1	0	-1	0	1
Tangent	0	∞	0	∞	0

For the angles 30° and 60° consider the following figures.



The Δ ABC is an equilateral triangle of side 2 units

From the figure, $\cos 30^0 = \frac{\sqrt{3}}{2}$

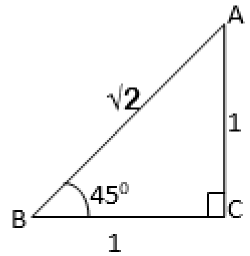
$\text{Sin } 30^0 = \frac{1}{2}$

$\text{Cos} 60^0 = \frac{1}{2}$

$\text{Sin } 60^0 = \frac{\sqrt{3}}{2}$

$\tan 30^0 = \frac{\sqrt{3}}{3}$ and $\tan 60^0 = \sqrt{3}$

For the angle 45^0 consider the following triangle



The Δ ABC has the sides AC = BC= 1 and AB = √2

Now Cos 45°=$\frac{1}{\sqrt{2}}$, sin 45° = $\frac{1}{\sqrt{2}}$ and tan 45°= $\frac{1}{1}$ =1

So Cos45^0 = sin45^0 = $\frac{1}{\sqrt{2}}$ and tan 45°= 1

The following table summarizes the Cosine, Sine, and tangent of the angle 30^0 , 45^0 and 60^0

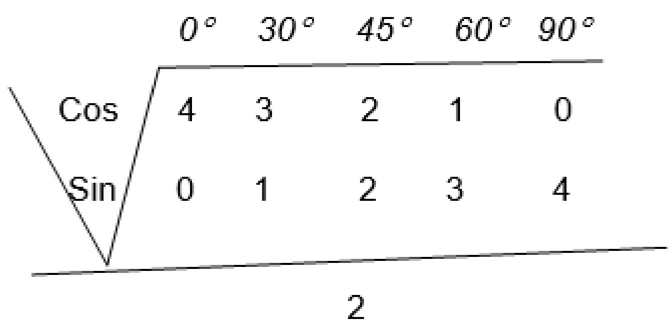
Angle	30^0	45^0	60^0
Sine	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tangent	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Remember tanθ = $\frac{Sin\theta}{Cos\theta}$

So tan60^0 for example is given by $\frac{\sin 60°}{\cos 60°}$ **=** $\frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

Therefore tan 60° = $\sqrt{3}$

NB: The following figure is helpful to remember the trigonometrical ratios of special angles from 0°to 90°



If we need the sines of the above given angles for examples, we only need to take the square root of the number below the given angle and then the result is divided by 2.

Eg. Sin 60°, here the number below 60° is 3 so the square root of 3 is $\sqrt{3}$

And Sin 60° $=\frac{\sqrt{3}}{2}$.

Example 7

Find the sine,cosine and tangents of each of the following angles

a. -135^0

b. 120^0

c. 330^0

Solution

a) $-135^0 = 360° - 135° = 225° = -45°$

So $\sin(-135°) = -\sin 225 = -\sin 45°$

$\sin(-135°) = -\frac{\sqrt{2}}{2}$

$\cos(-135°) = \frac{\sqrt{2}}{2}$

$\therefore \cos(-135) = \frac{\sqrt{2}}{2}$

$\tan(-135°) = \frac{\sin(-135°)}{\cos(-135°)} = -\frac{\sqrt{2}}{2} \div -\frac{\sqrt{2}}{2} = 1$

$\therefore \tan(-135°) = 1$

b) $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\frac{1}{2}$

$\therefore \cos 120° = \frac{-1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

c) $\sin 330^0 = -\sin 30^0 = -\frac{1}{2}$

$\cos 330^0 = \cos 30^0 = \frac{\sqrt{3}}{2}$ and $\tan 330° = -\tan 30° = \frac{-1}{\sqrt{3}}$

Example 8

Find the value of θif Cos θ= -½ and θ≤ θ≤ 360°

Solution

Since Cos θis – (ve), then θlies in either the second or third quadrants,

Now $-\cos(180 - \theta = -\cos(\theta + 180^0) = -\frac{1}{2} = -\cos 60^0$

So θ= 180^0-60^0 = 120^0 or θ= 180^0 + 60^0 = 240^0

θ= 120^0 0r θ=240^0

Example 9

Consider below

Evaluate $\frac{\tan 60^\circ \sin 30^\circ}{\cos 45^\circ}$

Solution:

$$\frac{\tan 60^\circ \sin 30^\circ}{\cos 45^\circ} = \frac{\sqrt{3}\times\frac{1}{2}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\therefore \frac{\tan 60^\circ \sin 30^\circ}{\cos 45^\circ} = \frac{\sqrt{3}}{\sqrt{2}} \text{ or } \frac{\sqrt{6}}{2}$$

Exercise 2

Solve the Following.

1. find the sine and cosine of

   (a) 250^0 (b) -72^0 (c) -157^0 (d) 289^0

2. Find the angles whose trigonometrical ratios are given and they lie between 0^0 and 360^0

   (a) Sin A = 0.3456

   (b) Tan B = 0.432

   (c) Cos C = - 0.896.

3. Without using tables evaluates the following

(a) Sin 60^0 Cos 60^0 (b) $\frac{Sin(-30)^\circ tan45^\circ}{Cos30^\circ}$

(c) Tan 345^0 $\cos 75^0$ – sin 60^0

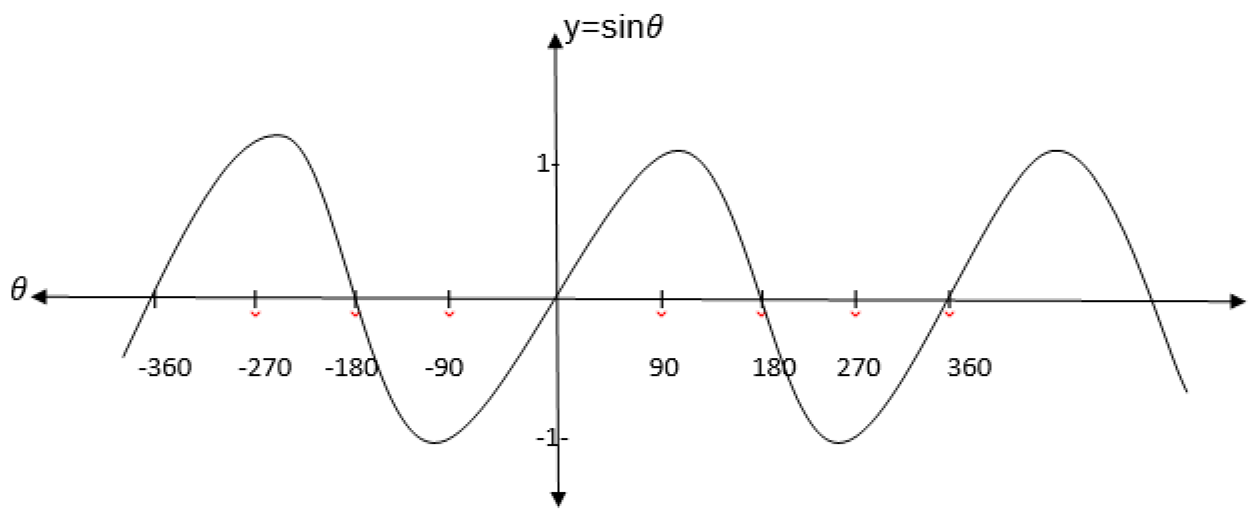
4. Given that sin120° = Cos 2θ, if θ is found in $\theta \leq \theta \leq 90^0$, find the value of tan$\theta$.

The Graphs of Sine and Cosine

Draw the graphs of sine and cosine

Consider the following table of value for y=sinθ where θranges from - 360°to 360°

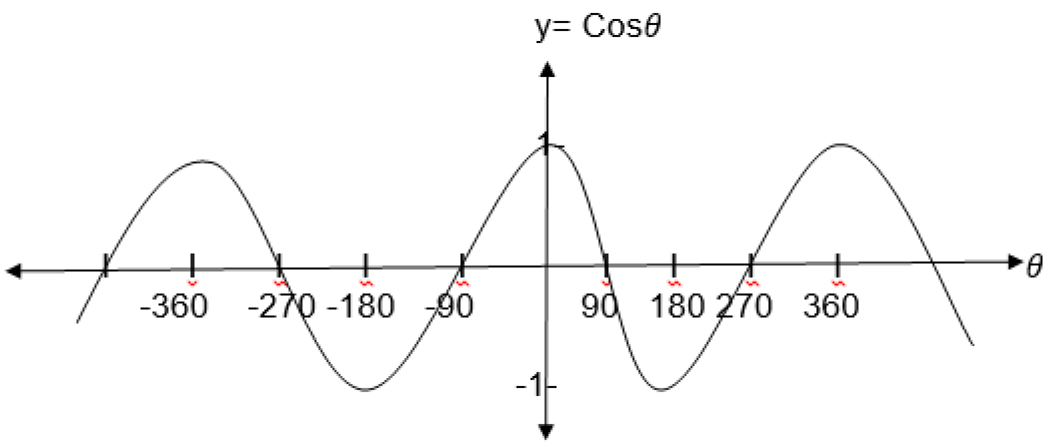
θ	-360°	-270°	-180°	-90	0°	90°	180°	270°	360°
Sin θ	0	1	0	-1	0	1	0	-1	0



For cosine consider the following table of values

θ	-360	-270	-180	-90	0	90	180	270	360
Cosθ	1	0	-1	0	1	0	-1	0	1

Its graph is as follows



From the graphs for the two functions a reader can notice that sinθand cosθboth lie in the interval -1 and 1 inclusively, that is $-1 \leq \sin\theta 1$ and $-1 \leq \cos\theta \leq 1$ for all values of θ.

The graph of y= tanθis left for the reader as an exercise

NB: $-\infty \leq \tan\theta \leq \infty$the symbol ∞means infinite

Also you can observe that both Sinθnd cosθrepeat themselves at the interval of

360°, which means $\sin\theta = \sin(\theta+360) = \sin(\theta+2x360^0)$ etc

and $\text{Cos}\theta = (\text{Cos}\theta+360^0) = \text{Cos}(\theta+2x360^0)$

Each of these functions is called a period function with a period 360^0

1. Usingtrigonometrical graphs in the interval $-360^0 \leq \theta \leq 360^0$

Find θsuch that

a. Sin= 0.4

b. Cos= 0.9

solution

Sin = 0.4

Then θ=-336^0, -204^0, 24^0, 156.

(b) Cos θ = 0.9

Then θ= -334^0, -26^0, 26^0, 334^0

Example 10

Use the graph of sinθto find the value ofθif

4Sinθ= -1.8 and -360^0 $\leq\theta\leq$360^0

Solution

4Sinθ= -1.8

Sinθ= -1.8÷4 = -0.45

Sinθ= -0.45

So θ= -153^0, -27^0, 207^0, 333^0

The graphs of sine and cosine functions

Interpret the graphs of sine and cosine functions

Example 11

Use thetrigonometrical function graphs for sine and cosine to find the value of

a. Sin (-40^0)

b. Cos (-40^0)

Solution

a. Sin (-40^0)= - 0.64

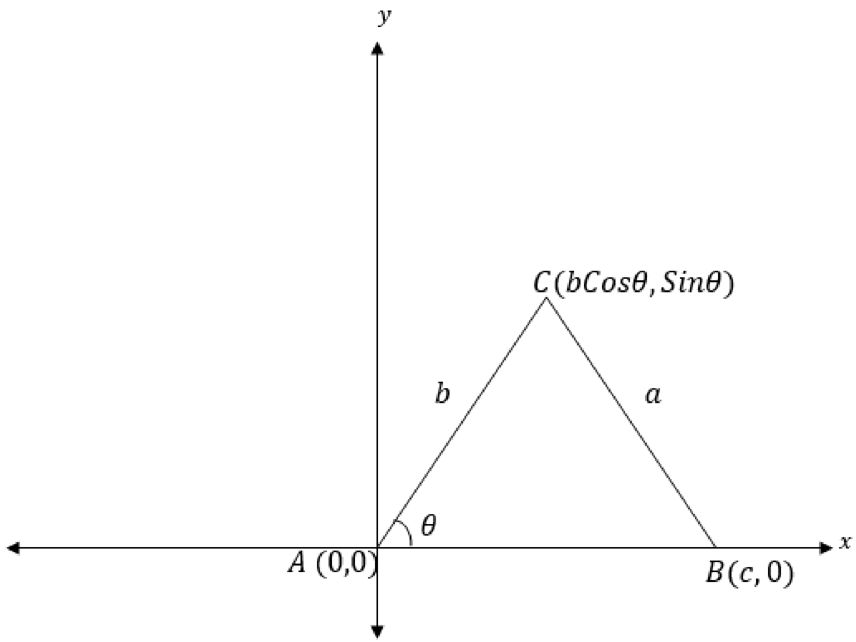
b. Cos (-40^0)= 0.76

Sine and Cosine Rules

The Sine and Cosine Rules

Derive the sine and cosine rules

Consider the triangle ABC drawn on a coordinate plane



From the figure above the coordinates of A, B and C are (0, 0), (c, 0) and(bCosθ, bSinθ) respectively.

Now by using the distance formula

$$BC = a = \sqrt{(\text{bCos}\theta - c)^2 + (\text{bSin}\theta)^2}$$

$$a = \sqrt{b^2cos^2\theta - 2bc\text{Cos}\theta + c^2 + b^2Sin^2\theta}$$

$$a^2 = b^2cos^2\theta - 2bc\text{Cos}\theta + c^2 + b^2Sin^2\theta$$

$$a^2 = b^2(cos^2\theta + Sin^2\theta) + c^2 - 2bc\text{Cos}\theta$$

But $(cos^2\theta + Sin^2\theta)$=1

$a^2 = b^2 + c^2 - 2bc\text{Cos}\theta$, θ corresponds to angle A

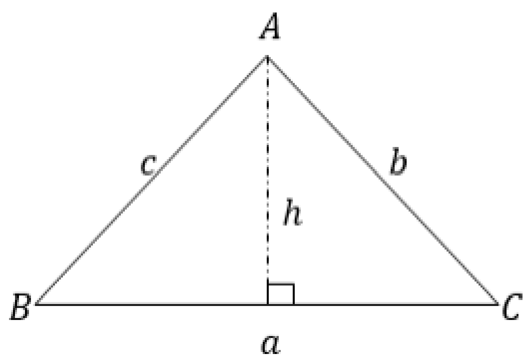
So $\boxed{a^2 = b^2 + c^2 - 2bc\text{Cos}A}$

Also $\boxed{b^2 = a^2 + c^2 - 2ac\text{Cos}B}$

And $\boxed{c^2 = a^2 + b^2 - 2ab\text{Cos}C}$

SINE RULE

Consider the triangle ABC below



From the figure above,

$\text{SinB} = \frac{h}{c}$ which means $h = c\text{SinB}$

Also $\text{SinC} = \frac{h}{b}$ *which means* $h = b\text{SinC}$

It follows that $h = c\text{SinB} = b\text{SinC}$, then $\frac{\text{SinB}}{b} = \frac{\text{SinC}}{c}$

It is easy to show that $a\text{SinB} = b\text{SinA}$, so $\frac{\text{SinB}}{b} = \frac{\text{SinA}}{a}$

Hence $\boxed{\frac{\text{SinA}}{a} = \frac{\text{SinB}}{b} = \frac{\text{SinC}}{C}}$

Note that this rule can be started as "In any triangle the side are proportional to the Sines of the opposite angles"

The Sine and Cosine Rules in Solving Problems on Triangles

Apply the sine and cosine rules in solving problems on triangles

Example 12

Find the unknown side and angle in a triangle ABC given that

a= 7.5cm

c= 8.6cm and C= 80°

Solution

By using sine rule, $\frac{\text{SinA}}{a} = \frac{\text{SinB}}{b} = \frac{\text{SinC}}{C}$

So $\frac{\text{SinA}}{7.5} = \frac{\text{Sin}80°}{8.6} = \frac{\text{SinB}}{b}$

$\frac{\text{SinA}}{7.5} = \frac{\text{Sin}80°}{8.6}$

$SinA = \frac{7.5\text{Sin}80°}{8.6} = 0.86$

Sin A= 0.86

A= 59.2°

But A+ B+C=180°

B = 180°-A-C

=180°-59.2°-80°

B= 40.8°

Also $\frac{\text{SinA}}{7.5} = \frac{\text{Sin}80°}{8.6} = \frac{\text{SinB}}{b}$

$\frac{\text{Sin}80°}{8.6} = \frac{\text{Sin}40.8°}{b}$

$\frac{8.6\text{Sin}40.8°}{\text{Sin}80°} = b$

$b = 5.7cm$

∴ A=59.2°, B= 40.8° and b= 5.7cm

Find the unknown sides and angle in a triangle ABC in which a= 22.2cmB= 86°and A= 26°

Solution

By sine rule

Sin A= sin B= Sin C

$$\frac{\text{SinA}}{a} = \frac{\text{SinB}}{b} = \frac{\text{SinC}}{c}$$

So $\frac{\text{Sin}26°}{22.2} = \frac{\text{Sin}86°}{b} = \frac{\text{SinC}}{c}$

But A + B +C = 180°

C= 180°-26°-86°

C= 68°

$$\frac{\text{Sin}26°}{22.2} = \frac{\text{Sin}86°}{b}$$

$$b = \frac{22.2 \times \text{Sin}86°}{\text{Sin}26°}$$

$$b = \frac{22.2\text{x}0.998}{0.438}$$

b = 50.52cm

Also $\frac{\text{Sin}68°}{c} = \frac{\text{Sin}26°}{22.2}$

$$c = \frac{22.2\text{Sin}68°}{\text{Sin}26°} = 46.95\text{cm}$$

∴ ***C= 68°, b = 50.52cm and c= 46.95cm***

Example 13

Find unknown sides and angles in triangle ABC

Where a=3cm, c= 4cm and B= 30°

Solution

By cosine rule,

$a^2 = b^2 + c^2 - 2bc\text{Cos}A$

$b^2 = a^2 + c^2 - 2ac\text{Cos}B$

$c^2 = a^2 + b^2 - 2ab\text{Cos}C$

b=? , A=? and C=?

Now $b^2 = 3^2 + 4^2 - (2 \times 3 \times 4)\text{Cos}30°$

$b^2 = 25 - 24 \times 0.866$

$b^2 = 4.215$

b = $2.05cm$

$c^2 = a^2 + b^2 - 2ab\text{Cos}C$

$4^2 = 3^2 + 2.05^2 - 2 \times 2.05 \times 3\text{Cos}C$

16= $9 + 4.2025 - 12.3\text{Cos}C$

$\frac{16-9-4.2025}{-12.3} = \text{Cos}C$

$\text{Cos}C$ = -0.22744

$C = 103.1°$

Also $a^2 = b^2 + c^2 - 2bc\text{Cos}A$

$3^2 = 2.05^2 + 4^2 - 2 \times 2.05 \times 4\text{Cos}A$

$3^2 - 2.05^2 - 4^2 = -2 \times 2.05 \times 4\text{Cos}A$

$-11.2025 = -16.4\text{Cos}A$

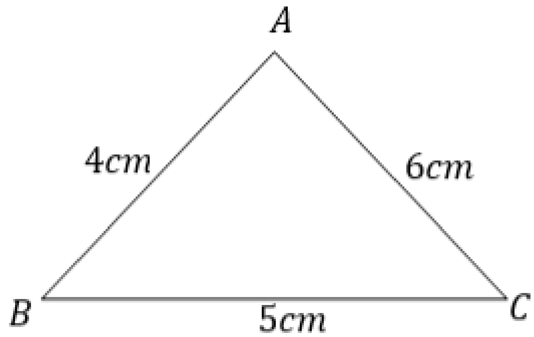
$\text{Cos}A = \frac{-11.2025}{-16.4} = 0.683$

A= 46.9°

$\therefore$ $\boldsymbol{b = 2.05cm, A = 46.9° \ and}$ **C= *103.1*°**

Example 14

Find the unknown angles in the following triangle



Solution

Using Cosine rule,

$a^2 = b^2 + c^2 - 2bc\cos A$

$5^2 = 6^2 + 4^2 - 2 \times 6 \times 4\cos A$

$25 = 36 + 16 - 48\cos A$

$25 - 36 - 16 = -48\cos A$

$-27 = -48\cos A$

$CosA = \frac{-27}{-48} = \frac{27}{48}$

$CosA = 0.5625$

$A = 55.8°$

Again $6^2 = a^2 + c^2 - 2acCosB$

$6^2 = 5^2 + 4^2 - 2 \times 5 \times 4Cos\ B$

$6^2 = 5^2 + 40Cos\ B$

$-5 = -40CosB$

$CosB = 0.125$

$B = 82.8°$

Also $C^2 = a^2 + b^2 - 2abCosC$

$4^2 = 5^2 + 6^2 - 5 \times 6 \times 2CosC$

$16 = 25 + 36 - 60CosC$

$-45 = -60CosC$

$CosC = \frac{45}{60} = 0.75$

$C = 41.4°$

$\therefore$ ***A=55.8°, B=82.8° and C=41.4°***

Exercise 3

1. Given thata=11cm, b=14cm and c=21cm, Find the Largest angle of ΔABC

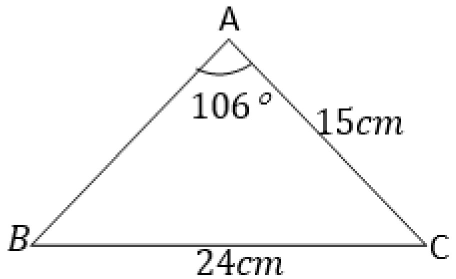
2. If ABCD is a parallelogram whose sides are 12cm and 16cm what is the length of the diagonal AC if angle B=119°?

3. A and B are two ports on a straight Coast line such that B is 53km east of A. A ship starting from A sails 40km to a point C in a direction E65°N. Find:

a. The distance a of the ship from B

b. The distance of the ship from the coast line.

4. Find the unknown angles and sides in the following triangle.



5. A rhombus has sides of length 16cm and one of its diagonals is 19cm long. Find the angles of the rhombus.

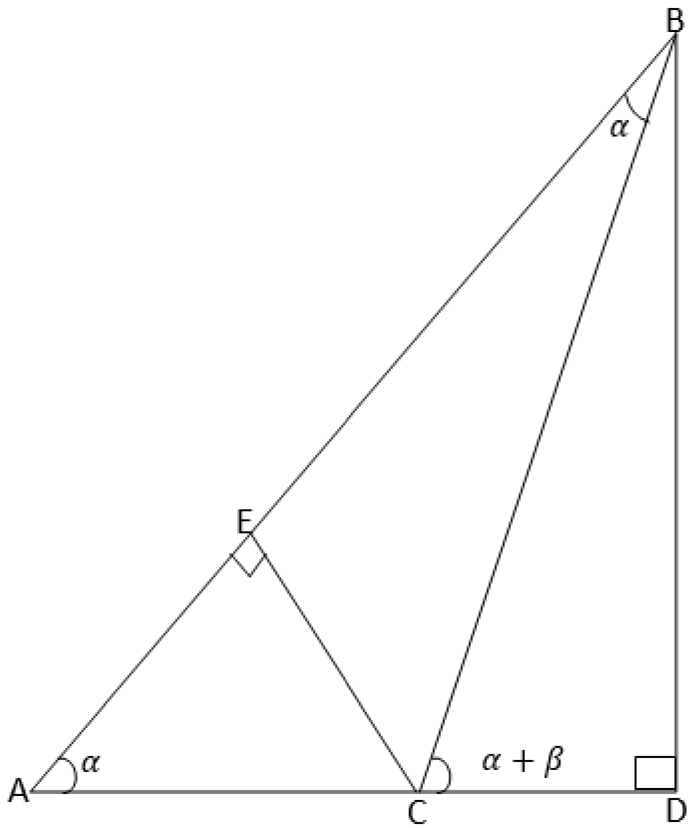
Compound Angles

The Compound of Angle Formulae or Sine, Cosine and Tangent in Solving Trigonometric Problems

Apply the compound of angle formulae or sine, cosine and tangent in solving trigonometric problems

The aim is to express Sin (α±β) and Cos (α±β) in terms of Sinα, Sinβ, Cosαand Cosβ

Consider the following diagram:



From the figure above

From ΔBCD

$\text{Sin}\,(\alpha+\beta) = \frac{BD}{Bc}$ …………………… (1)

From the same figure $\text{Sin}\alpha = \frac{BD}{AB} = \frac{EC}{AC}$,

$\text{Cos}\alpha = \frac{AE}{AC} = \frac{AD}{AB}$, $\text{Sin}\beta = \frac{EC}{BC}$ and $\text{Cos}\beta = \frac{EB}{BC}$

Now $\text{Sin}\,(\alpha+\beta) = \frac{BD}{BC} = \frac{AB\,Sin\alpha}{BC}$

$\text{Sin}\,(\alpha+\beta) = \frac{(AE+EB)Sin\alpha}{BC}$

$= \frac{AE}{BC}\text{Sin}\alpha + \frac{EB}{BC}\text{Sin}\alpha$

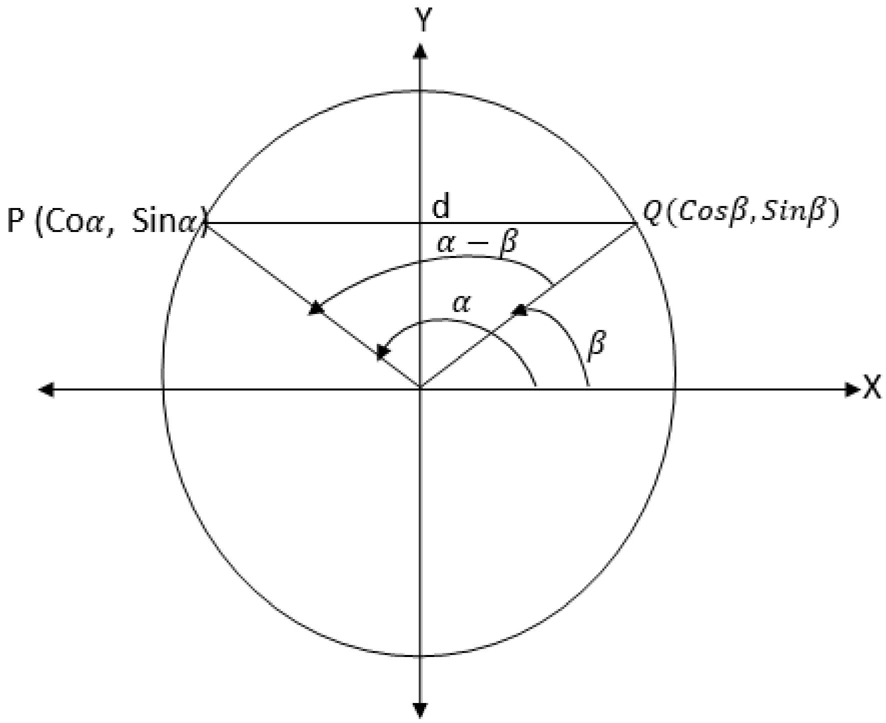
$= \left(\frac{AE}{BC}\right) \times \left(\frac{Ac}{AC}\right) + \left(\frac{EB}{BC}\right)\text{Sin}\alpha$

But $\frac{AE}{AC} = \text{Cos}\alpha$, $\frac{EC}{BC} = \text{Sin}\beta$ and $\frac{EB}{BC} = \text{Cos}\beta$

It follows that $\text{Sin}\,(\alpha+\beta) = \text{Cos}\alpha\text{Sin}\beta + \text{Cos}\beta\text{Sin}\alpha$

$\therefore$ ***Sin*** $(\boldsymbol{\alpha}+\boldsymbol{\beta})$ **=** $\boldsymbol{Cos\alpha Sin\beta + Cos\beta Sin\alpha}$

For Cos(α±β) Consider the following unit circle with points P and Q on it such that OP,makes angleα with positive x-axis and OQ makes angle βwith positive x-axes.



From the figure above the distance d is given by

d^2=2-2Cos $(\alpha-\beta)$.

Therefore 2-2(CosαCosβ+SinSαSinβ) = 2-2Cos $(\alpha-\beta)$

Or Cos $(\alpha-\beta)$ = CosαCosβ+SinSαSinβ

∴ **Cos $(\alpha-\beta)$ = CosαCosβ+SinSαSinβ**

Because cosine and Sine are even and Odd function respectively, then we can find cos $(\alpha+\beta)$ and Sin (($\alpha-\beta$) as follows:-

$\alpha+\beta = \alpha-(-\beta)$

So Cos $(\alpha+\beta)$ = CosαCos$(-\beta)$+SinSαSin$(-\beta)$

But Cos (-β) = Cos $(-\beta)$ and Sin (-β) =-sinβ.

Then *Cos $(\alpha+\beta)$ = CosαCosβ-SinαSinβ*

∴ **Cos $(\alpha+\beta)$ = CosαCosβ-SinSαSinβ**

Again $\alpha - \beta = \alpha + (-\beta)$

So Sin $(\alpha - \beta)$ = Sin $(\alpha + (-\beta))$

=CosαSin$(-\beta)$+Cos$(-\beta)$ Sinα

= SinαCosβ- CosαSinβ

∴ **Sin $(\alpha - \beta)$= SinαCosβ- CosαSinβ**

In general

Cos $(\alpha + \beta)$ = CosαCosβ-SinSαSinβ…………………… (i)

Cos $(\alpha - \beta)$ = CosαCosβ+SinSαSinβ……………….. (ii)

Sin $(\alpha + \beta)$ = CosαSinβ+CosβSinα………….. ……. (iii)

Sin $(\alpha - \beta)$= SinαCosβ- CosαSinβ………………….. (iv)

Example 15

1. Without using tables find the value of each of the following:

a. Sin 75°

b. Cos 105

Solution:

(a) Sin 75°

75°= 45°+30°(sum of special angles)

Sin75°= Sin (45°+30°)

=Sin45°Cos30°+Sin30°Cos45°

Sin 75°= $\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{4}(\sqrt{6}+\sqrt{2})$

$\therefore$ ***Sin 75°*** **=** $\mathbf{\frac{1}{4}(\sqrt{6}+\sqrt{2})}$

(b) Cos105°

105°= 60°+45°(sum of special angles)

Cos (105°) = Cos (60°+45°)

= Cos60°Cos45°- Sin60°Sin45°.

$=\frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{4}(\sqrt{2}-\sqrt{6})$

$\therefore$ ***Cos105°*** **=** $\mathbf{\frac{\sqrt{2}-\sqrt{6}}{4}}$

Example 16

Find:

a. Sin150°

b. Cos 15°

150°=90°+60°

Sin150°=Sin (90°+60°)

=Sin90°Cos60°+Sin60°Cos90°

$=1\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times 0=\frac{1}{2}$

$\therefore$ ***Sin150°=*** $\mathbf{\frac{1}{2}}$

(c) Cos15° = 45°-30° or 60°- 45°

Cos15°=Cos45°Cos30°+Sin45°Sin30°

$=\frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}=\frac{1}{4}(\sqrt{2}+\sqrt{6})$

$\therefore$ ***Cos15°=*** $\mathbf{\frac{1}{4}(\sqrt{2}+\sqrt{6})}$

Exercise 4

1. Withoutusing tables, find:

a. Sin 15°

b. Cos 120°

2. Find Sin 225° from (180°+45°)

3. Verify that

a. Sin 90° = 1 by using the fact that 90°=45°+45°

b. Cos90°=0 by using the fact that 90°=30°+60°

4. Express each of the following in terms of sine, cosine and tangent of acute angles.

a. Sin107°

b. Cos300°

5. By using the formula for Sin (A-B), show that Sin (90°-C)=Cos C