

PROBABILITY

Normally we are living in the world full of uncertainties.

For example when two equally strong foot ball teams play a match it is not easy to predicate the outcome of the game. Also for a pregnant woman it is not easy to predict what will be the sex of the born. Under such uncertainties the theory of probability is applied.

Definition; Probability is a branch of mathematics which deals with and shows how to measure the occurrence of events in daily life. Or it can simply be defined as a measure of chances.

Probability of an Events

The Probability of an Event Through Experiments

Determine the probability of an event through experiments

Probability set (s)

Definition: Probability set is the set of all outcomes/results from the experiment being performed.

For example when tossing once a fair coin the expected outcomes are either head(H) or tail(T) to be shown up.

In this case the probability set is

$$S = \{H, T\}$$

Also if a fair die is tossed once what is expected to show up is only one number among the six numbers, that is 1,2,3,4,5,6.

Now the probability set is

$$S = \{1, 2, 3, 4, 5, 6\}.$$

An event (E); An event is a specified outcome from the probability set.

For example a head (H) in the experiment of tossing a fair coin is an event and it is a sub set of the probability set,

Thus, $S = \{H, T\}$ and $E = \{H\}$.

An event may or may not occur. For example if the event that a head occurs in tossing a fair coin once but a tail occurs instead , then the event did not occur and it is denoted by E' which is the complement of E .

So if $S = \{H, T\}$ and the event $E = \{H\}$, then E' is the event that H does not occur, hence $E' = \{T\}$.

NB: A probability set is also called a sample space

Example 1

1. A die is tossed once and the results are recorded. Find

- The probability set (sample space)
- The event that an even number occurs.
- The event that an even number does not occur.

Solution;

- The sample space $S = \{1, 2, 3, 4, 5, 6\}$
- The event that an even number occurs is $E = \{2, 4, 6\}$.
- The event that an even number does not occur is $E' = \{1, 3, 5\}$

Example 2

Give the probability set of the experiment of selecting even numbers less than 20.

Solution

$S = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$.

Example 3

Give the probability set of not selecting an even number from a set of counting numbers less than 9.

Solution

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$E = \{2, 4, 6, 8\}$$

$$\text{So } E' = \{1, 3, 5, 7\}$$

where E is the event of selecting an even number and E' is the event of not selecting even number less than 9.

Exercise 1

1. Write the probability set of each of the following experiments:

- a. A die is tossed and the face showing up is read.
- b. A friend is asked for the month of his birth.
- c. The sex of a human being is asked.
- d. A card is drawn from a box containing five cards bearing the numerals 2, 4, 6, 8 and 10.

2. Write in set notation the elements of the following events:

- a. A fair die is rolled and the number obtained is greater or equal to 5.
- b. A prime number between 20 and 40 is chosen.

3. Write in set notation the elements of the event of not choosing an even number between 25 and 55

Experimental Results in Relation to Real Life Occurrences

Interpret experimental results in relation to real life occurrences

For example when tossing once a fair coin the expected outcomes are either head(H) or tail(T) to be shown up.

In this case the probability set is

$$S = \{H, T\}$$

Also if a fair die is tossed once what is expected to show up is only one number among the six numbers, that is 1,2,3,4,5,6.

Now the probability set is

$$S = \{1, 2, 3, 4, 5, 6\}.$$

The Formula for Finding the Probability of an Event

Write the formula for finding the probability of an event

Probability of an event:

Definition: The probability of an event is the ratio between the number of times the event has occurred to the total number of experiments that have been done.

If $P(E)$ Is the probability of the event E , then

$$P(E) = \frac{\text{Number of times } E \text{ has occurred}}{\text{Total number of times the experiment has been done}}$$

Or $P(E) = \frac{n(E)}{n(S)}$, where $P(E)$ is the probability of an event E , $n(E)$ is the number of elements in (E) which is actually the number of times E has occurred and $n(S)$ is the number of elements in the sample space which is the total number of possible outcomes of the experiment.

NB: $\frac{n(E)}{n(S)}$ is the ratio called the observed relative frequency of the event which is the probability of an event E to occur.

Also the probability found by experimenting is referred to as experimental probability.

The Formula to Calculate the Probability of an Event

Apply the formula to calculate the probability of an event

Example 4

A drawing pin was tossed 1000 times. The number of tosses where the pin fell flat was 563. Calculate the probability that when such a pin is tossed, it will fall flat.

Solution:

$$P(E) = \frac{n(E)}{n(S)} = \frac{563}{1000} = 0.563$$

The probability is 0.563.

Example 5

5% of torch bulbs manufactured by a certain factory were defective. What is the probability that when a bulb from that factory is tested it will be defective?

Solution:

$$P(E) = 5\% = 5/100 = 0.05$$

Note that the probability of an event is defined under the condition that every outcome has an equal chance of occurring as other outcomes. Here we say the outcomes are equally likely or equiprobable.

Words like random selection, fair die and a fair coin are used mean that the choice is impartial (unbiased)

$$S = \{H, T\}, E = \{H\}$$

$$\text{From } P(E) = \frac{n(E)}{n(S)}, P(E) = 1/2 = 0.5$$

$$P(E) = \frac{1}{2}$$

Therefore the probability of head to occur is $\frac{1}{2}$ or 0.5

Example 6

A piece of chalk is picked from a box containing 5 identical pieces two of which are red and the remaining are white. Find the probability that the piece of chalk picked is red

Solution;

$$S = (r_1, r_2, w_1, w_2, w_3)$$

Where r_n and w_n means the colors red and white respectively.

$$E = (r_1, r_2)$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}, \text{ therefore the probability that the piece of chalk picked is red is } \frac{2}{5}$$

Example 7

Find the probability that a king appears in a drawing a single card from an ordinary deck of 52 cards.

Solution:

$n(S) = 52$ and $n(E) = 4$ since there are 4 kings out of 52 cards.

$$\text{So } P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

So the probability is $\frac{1}{13}$.

NB. E' is the event that E does not occur, that is E' is the complement of E .

From the set theory $n(U) = n(A) + n(A')$ where U is the universal set and A is any set (a subject) found in U .

Therefore $n(S) = n(E) + n(E')$

But,

$$P(E) = \frac{n(E)}{n(S)} \text{ and } P(E') = \frac{n(E')}{n(S)}$$

$$\begin{aligned} \text{Then } P(E) + P(E') &= \frac{n(E)}{n(S)} + \frac{n(E')}{n(S)} \\ &= \frac{n(E) + n(E')}{n(S)} = \frac{n(S)}{n(S)} = 1, \end{aligned}$$

\therefore

$$P(E) + P(E') = 1$$

Generally for any event E, $0 \leq P(E) \leq 1$

Normally an **impossible event** or the event which cannot occur under any condition is denoted by \emptyset in set intention. And because $n(\emptyset) = 0$, then the probability of an event that can't occur is 0.

Example 8

What is the probability of not getting an even number when a fair die is tossed?

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{2, 4, 6\}$$

$$E' = \{1, 3, 5\}$$

$$P(E') = \frac{n(E')}{n(S)} = 3/6 = 1/2$$

The probability of not getting an even number is 1/2 or 0.5,

Alternatively,

$$P(E) = \frac{n(E)}{n(S)} = 3/6$$

$$\text{But } P(E) + P(E') = 1$$

$$\begin{aligned} P(E') &= 1 - P(E) \\ &= 1 - 3/6 \end{aligned}$$

$$P(E') = 1/2 = 0.5$$

Example 9

What is the probability of selecting a green ball from the box containing red and green balls if the probability of selecting red ball is 1/4?

Solution:

$$\text{From } P(E) + P(E') = 1$$

The probability of selecting a green ball is the probability of not selecting a red ball.

$$\text{Now } P(E') = 1 - P(E)$$

$$\text{But } P(E) = 1/4$$

$$P(E') = 1 - 1/4$$

$$P(E') = 3/4$$

The probability of selecting a green ball is 3/4.

Example 10

When tossing a die what is the probability of getting a number greater or equal to 1?

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{1, 2, 3, 4, 5, 6\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1 \quad \therefore \text{the probability of getting a number greater or equal to 1 is 1.}$$

Note: In the example number 8, one numeral between 1 and 6 inclusively must show up when the die is rolled. Therefore the probability of an event that must happen is 1.

Exercise 2

For practice.

1. Find the probability of choosing a number divisible by 2 from a set of numbers between 20 and 45.
2. The total number of red and white pieces of chalk that are contained in a box is 20. How many pieces of white chalk are in the box if the probability of choosing a red piece of chalk is 2/5, given that the pieces are identical?
3. What is the probability that a month selected at random from the twelve months of the year will have 31 days?

4. A survey conducted at certain maternity ward showed that 60% of children born were female. What is the probability that Moses' child, who was born in that ward is a male?

5. A die was tossed 100 times, the six numbers with their frequency of occurrence were recorded in the following table:

Number	1	2	3	4	5	6
Frequency	14	17	20	18	15	16

Find the probability that:

- (a) A number 5 appears
- (b) A number less than 4 appears
- (c) A non – even number appears

Combined Events

Experiments of Two Combined Events

Perform experiments of two combined events

If two or more simple events may occur or take place at the same time then the events are combined events.

For instance when the experiment of tossing two coins at the same time is done, then the event of interest can't simply be determined.

Let $E = \{\text{Obtaining two heads}\}$

In this case there are two simple events which are obtaining the head on the first coin and obtaining the head on the second coin.

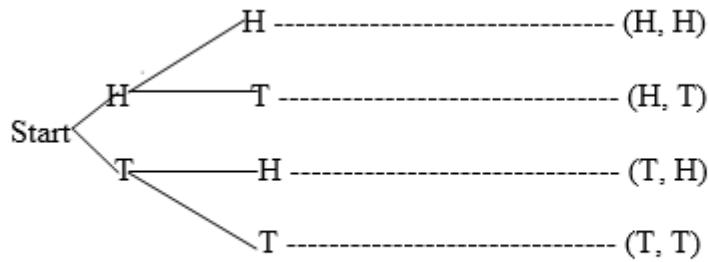
So $E_1 = \{\text{Obtaining the head on the first coin}\}$

$E_2 = \{\text{Obtaining the head on the second coin}\}$

Drawing a Tree Diagram of Combined Events

Draw a tree diagram of combined events

The event E can be found by using what is referred to as a tree diagram.



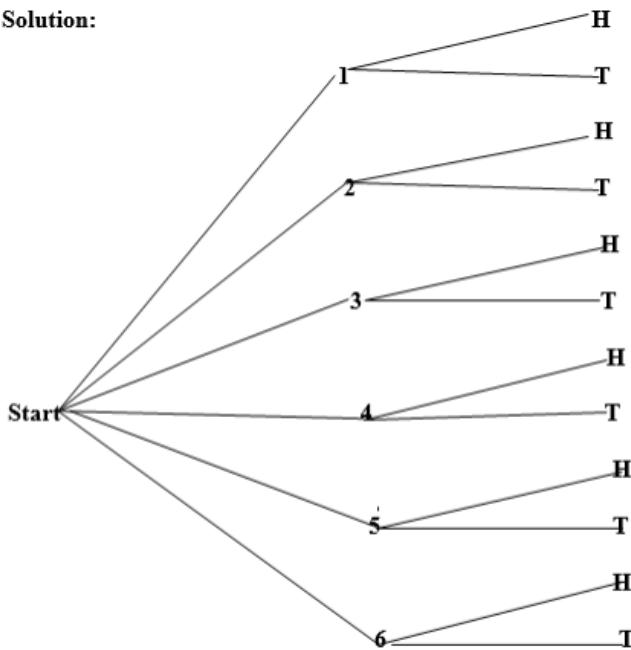
Note that the events are read along the paths from the starting point of events 1, 2, 3 and 4.

Therefore, $S = \{(H, H), (H, T), (T, H), (T, T)\}$ is the sample space.

Example 11

A die and Coin are tossed together. Draw a tree diagram to find the Sample space and hence determine the probability that a head and a number less than 3 occurs.

Solution:



From the tree diagram above, the sample space S is given by

$$S = \{(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)\}$$

$$E = \{\text{Head and a number less than 3}\}, E = \{(1, H), (2, H)\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

$$\therefore P(E) = \frac{2}{12} = \frac{1}{6}$$

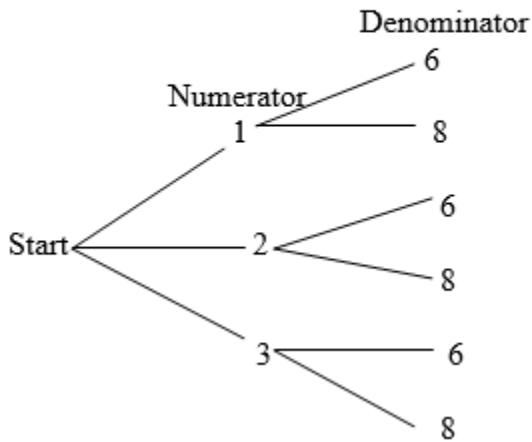
Example 12

A fraction is written by selecting the numerator from the digits 1, 2, 3 and the denominator from the digits 6, 8.

- a. Draw a tree diagram to find the sample space of this experiment.
- b. Find the probability that a the fraction written is less than $\frac{1}{2}$

Solution:

(a)



$$S = \left\{ \frac{1}{6}, \frac{1}{8}, \frac{2}{6}, \frac{2}{8}, \frac{3}{6}, \frac{3}{8} \right\}$$

$$(b) E = \left\{ \frac{1}{6}, \frac{1}{8}, \frac{2}{6}, \frac{2}{8}, \frac{3}{8} \right\}$$

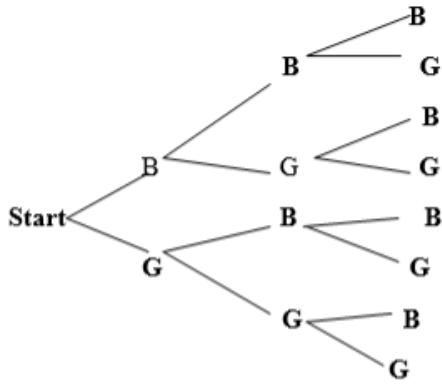
$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{6}$$

$$\therefore P(E) = \frac{5}{6}$$

Example 13

In a family of 3 children what is the probability that

- All are girls
- At least two are boys

Solution

$$S = \{(B, B, B), (B, B, G), (B, G, B), (B, G, G), (G, B, B), (G, B, G), (G, G, B), (G, G, G)\}$$

(a) $E = \{G, G, G\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

$$\therefore P(E) = \frac{1}{8}$$

(b) $E = \{(B, B, B), (B, B, G), (B, G, B), (G, B, B)\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2} \quad \therefore P(E) = \frac{1}{2}$$

Example 14

Three coins are tossed simultaneously. Find the probability that

- 3 heads appear
- 2 tails and one head appear

(a) $E = \{(H, H, H)\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{8} \quad \therefore \text{ the probability that 3 heads appear is } \frac{1}{8}.$$

(b) $E = \{(H, T, T), (T, H, T), (T, T, H)\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

\therefore The probability that 2 tails and one head appear is $\frac{3}{8}$

Exercise 3

1. If two digitsnumeral is written choosing ten's digits from the set {1, 2, 3, 4,} and the unit's digit from {5,6} what is the probability that a number greater than 20 will appear?
2. A pair of dice is tossed. Find the probability that the sum of the two numbers obtained is
 - a. At least 8
 - b. at most 1
 - c. Exactly 6
3. In a family with two children, what is the probability that:
 - a. Both are boys
 - b. At least one is a boy
4. A die and two coins are tossed at the same time find the probability that
 - a. A number 6 and two heads will appear.
 - b. A number less than 4, a head and tail will appear.
 - c. A number multiple of 2 and two tails will appear.

The Probability of Two Combined Events using the Formula

Find the probability of two combined events using the formula

Mutually Exclusive Events;

Two or more events are said to be mutually exclusive if the occurrence of one event hinders the occurrence of the other. This means that for mutually exclusive events, only one event may occur at a time, e.g., it is impossible for two numbers say 1 and 6 on a single die to show up for one tossing.

Therefore if A and B are two events, then the probability of A or B is given by

$$P(A \cup B) = P(A) + P(B)$$

Example 15

If in a class there are 34 students instead of 35 and Issa, Anna, Eliza and Juma apply for the one chance remaining what is the probability that either Anna or Juma will be chosen?

Solution: Since there are 4 people applying for the chance, then,

$$P(\text{Anna}) = P(\text{Issa}) = P(\text{Eliza}) = P(\text{Juma}) = \frac{1}{4}$$

$$\text{But } P(A \text{ or } B) = P(A) + P(B)$$

$$\text{So } P(\text{Anna or Juma}) = P(\text{Anna}) + P(\text{Juma}),$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

\therefore The probability that either Juma or Anna will be chosen is $\frac{1}{2}$.

Example 16

Find the probability that an even or an odd number which is greater than 1 occurs when a die is tossed once.

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

$$E_1 = \{2, 4, 6\}.$$

$$E_2 = \{3, 5\}$$

$$P(E_1, \text{ or } E_2) = P(E_1) + P(E_2)$$

$$P(E_1) = \frac{1}{2} \text{ and } P(E_2) = \frac{2}{6} = \frac{1}{3}$$

$$P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

∴ Find the probability that an even or an odd number which is greater than 1 occurs is $\frac{5}{6}$

Example 17

The following table shows years of experience for plumbers in a builders company.

Year of Experience	Number of Builders	Probability
0 - 2	6	1/8
3 - 5	12	1/4

6 - 8	8	1/6
More than 8	22	11/24
Total	48	

If the foreman wants to select a plumber for a specific assignment, what is the probability that

Solution:

$$P(6 \text{ or more}) = P(6 \text{ to } 8) + P(\text{More than } 8)$$

$$= 1/6 + 11/24 = \frac{4+11}{24}$$

$$P(6 \text{ or more}) = \frac{15}{24} = \frac{5}{8}$$

Therefore the probability of selecting a plumber with 6 or more years of experience is $\frac{5}{8}$

Independent Events

Definition: Two or events are said to be independent events if the occurrence of one event does not affect the occurrence of other event(s)

For example when a die and a coin are tossed together, the occurrence of a tail on the coin does not hinder the occurrence of the number 5 on the die.

Now, if A and B are independent events, then the probability of A and B is given by:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

Example 18

A die and coin are tossed. Find the probability that a number greater than 4 appears on the die and a tail appears on the coin:

Solution

For a die, $S = \{1, 2, 3, 4, 5, 6\}$

$$E_1 = \{5, 6\}$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

For the coin, $S = \{H, T\}$

$$E_2 = \{T\}$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{2}$$

$$\text{So } P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Therefore the probability is $\frac{1}{6}$.

Note that the same answer can be obtained by using a tree diagram.

Example 19

A box contains 9 oranges, 7 mangoes and 2 lemons. A fruit is drawn from the box and then replaced. Another draw is made. What is the probability that both fruits drawn are mangoes.

Solution:

$$n(S) = 18 \quad n(\text{mango}) = 7$$

$$P(\text{1st fruit is a mango}) = 7/18$$

$$P(\text{2nd fruit is a mango}) = 7/18$$

$$P(\text{1st and 2nd fruits are mangoes}) = P(\text{1st fruit being a mango}) \times P(\text{2nd fruit being a mango})$$

$$= \frac{7}{18} \times \frac{7}{18} = \frac{49}{324}$$

∴ The probability that both fruits drawn are mangoes is $\frac{49}{324}$.

Example 20

The probability that a man and his wife will be alive for 50 years are $\frac{3}{10}$ and $\frac{1}{3}$ respectively. Find the probability that

Solution:

$$n(S) = 18 \quad n(\text{mango}) = 7$$

$$P(\text{1st fruit is a mango}) = \frac{7}{18}$$

$$P(\text{2nd fruit is a mango}) = \frac{7}{18}$$

$$P(\text{1st and 2nd fruits are mangoes}) = P(\text{1st fruit being a mango}) \times P(\text{2nd fruit being a mango}) \\ = \frac{7}{18} \times \frac{7}{18} = \frac{49}{324}$$

∴ The probability that both fruits drawn are mangoes is $\frac{49}{324}$.

Exercise 4

Self test.

1. A coin is tossed and a card is drawn from an ordinary pack of 52 cards. Find the probability that an ace is drawn and a head is obtained on the coin (There 4 aces in a pack of cards)
2. Two numbers are selected from the integers 1 to 11 inclusively, repetition being allowed. Find the probability that (a) Both prime (b) Both are powers of 2
3. In the village, the probability that a man selected at random on a Sunday morning is carrying more than 30kg is 0.7. Find the probability that;
 - a. Two men selected at random on a Sunday morning is carrying more than 30kg
 - b. Three men selected at random are all carrying more than 30kg
4. A letter is chosen from the word “random” What is the probability that it is an n or d?
5. (a) What does it mean by saying that the probability of an event is (i) 0 (ii) 1 (b) Give two examples of impossible events.

The Knowledge of Probability to Determine the Occurrence of Events in Real Life Situation

Apply the knowledge of probability to determine the occurrence of events in real life situation

Probability is an area of mathematics which we use all the time in daily life – and usually without thinking about it. While many aspects are very intuitive, probabilities may be different for different people. I might estimate that the chance of rain is 70%, while a meteorologist with detailed weather data might say the chance of rain is 64.2%.

Activity 1

Apply the knowledge of probability to determine the occurrence of events in real life situation